

Name: _____

Date: _____

A2 CC1 Remainder Theorem

In the last lesson, we saw how two polynomials, when divided, resulted in another polynomial and a remainder. The remainder has a remarkable property in certain types of division. We will explore this relationship in the first exercise.

Exercise #1: Consider each of the following scenarios where we have $\frac{p(x)}{x-a}$. In each case, simplify the division using polynomial long division and then evaluate $p(a)$.

(a) $\frac{x^2 - 8x + 18}{x - 2}$

$$p(x) = x^2 - 8x + 18 \Rightarrow p(2) =$$

(b) $\frac{x^2 - 2x - 25}{x - 7}$

$$p(x) = x^2 - 2x - 25 \Rightarrow p(7) =$$

(c) $\frac{2x^2 + 11x + 11}{x + 3}$

$$p(x) = 2x^2 + 11x + 11 \Rightarrow p(-3) =$$

(d) $\frac{3x^2 + 7x - 20}{x + 4}$

$$p(x) = 3x^2 + 7x - 20 \Rightarrow p(-4) =$$

THE REMAINDER THEOREM

When the polynomial $p(x)$ is divided by the binomial $(x-a)$ then the remainder will always be $p(a)$. In other words:

$$\frac{p(x)}{(x-a)} = q(x) + \frac{p(a)}{x-a}$$

Exercise #2: If the ratio $\frac{x^2 - 11x + 22}{x-9}$ was placed in the form $q(x) + \frac{r}{x-9}$, where $q(x)$ is a linear function, then which of the following is the value of r ?

- (1) -3
- (2) 5
- (3) -9
- (4) 4

In the past, the remainder theorem was used primarily to aid in evaluating polynomials. These days it is the primary justification for telling if a linear expression is a factor of a polynomial.

Exercise #3: By definition $(x-a)$ is a factor of $p(x)$ if $\frac{p(x)}{x-a} = q(x)$, where $q(x)$ is another polynomial. What must be true of the remainder, $p(a)$, for $(x-a)$ to be a factor of $p(x)$? Explain.

Exercise #4: Determine if each of the following are factors of the listed polynomials by evaluating the polynomials.

- (a) Is $x-3$ a factor of $p(x) = x^2 - 11x + 24$?
- (b) Is $x+5$ a factor of $p(x) = 2x^2 + 9x - 2$?
- (d) Is $x+1$ a factor of $p(x) = x^3 - 7x^2 - 11x - 3$?
- (c) Is $x-5$ a factor of $p(x) = x^3 - x^2 - 19x - 10$?

THE REMAINDER THEOREM
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Which of the following is the remainder when the polynomial $x^2 - 5x + 3$ is divided by $(x - 8)$?
- (1) 107 (3) 3
- (2) 27 (4) 9
- _____
2. If the ratio $\frac{2x^2 + 17x + 42}{x + 5}$ is placed in the form $q(x) + \frac{r}{x + 5}$, where $q(x)$ is a polynomial, then which of the following is the correct value of r ?
- (1) -3 (3) 18
- (2) 177 (4) 7
- _____
3. When the polynomial $p(x)$ was divided by the factor $x - 7$ the result was $x + \frac{11}{x - 7}$. Which of the following is the value of $p(7)$?
- (1) -8 (3) 11
- (2) 7 (4) It does not exist
- _____
4. Which of the following binomials is a factor of the quadratic $4x^2 - 35x + 24$? Try to do this without factoring but by using the Remainder Theorem.
- (1) $x + 6$ (3) $x - 8$
- (2) $x - 4$ (4) $x + 2$
- _____
5. Which of the following linear expressions is a factor of the cubic polynomial $x^3 + 9x^2 + 16x - 12$?
- (1) $x + 6$ (3) $x - 3$
- (2) $x - 1$ (4) $x + 2$
- _____

6. Consider the cubic polynomial $p(x) = x^3 + x^2 - 46x + 80$.

(a) Using polynomial long division, write the ratio of $\frac{p(x)}{x-3}$ in **quotient-remainder form**, i.e. in the form

$q(x) + \frac{r}{x-3}$. Evaluate $p(3)$. How does this help you check your quotient-remainder form?

(b) Evaluate $p(5)$. What does this tell you about the binomial $x-5$?

(c) If $q(x) = \frac{p(x)}{x-5}$, then use polynomial long division to find an expression for the polynomial $q(x)$.

(d) Use your answer from (c) to **completely factor** the cubic polynomial $p(x)$. Besides $x=5$, what are its other zeroes?

7. For the cubic $x^3 + 7x^2 + 13x + 3$ has only one rational zero, $x = -3$. Divide to show that the remainder is zero when dividing the cubic by $x+3$. Then use the quadratic formula to find the other two (irrational) zeroes.